❶•❶ Functions, inverse functions & composite functions

Solutions

1. $\text{LHS} = (\cos 8x + \cos 2x)^2 + (\sin 8x + \sin 2x)^2$

$= (2\cos 5x \cos 3x)^2 + (2\sin 5x \cos 3x)^2$

$= 4\cos^2 3x (\cos^2 5x + \sin^2 5x)$

$= 4\cos^2 3x = \text{RHS}$

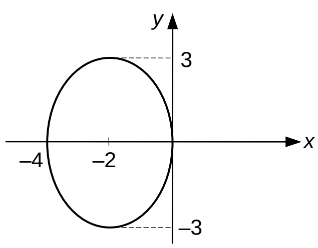
$4\cos^2 3x = 3$

$\cos 3x = \pm\frac{\sqrt{3}}{2}$

$3x = 30°, 150°, 210°, 330°, 390°, 510°...$

$x = 10°, 50°, 70°, 110°, 130°, 170°...$ **(ans)**

2.
(a)



$\frac{y^2}{9} + \frac{(x+2)^2}{4} = 1$

$r_f = (0, 3]$ **(ans)**

(b) Since $r_f \subseteq d_g$, gf is defined.

$r_{gf} = (-\infty, \ln 3]$ **(ans)**

3.
(a) Let $y = \ln(x-1)$

$x = e^y + 1$

$h^{-1} : x \to e^x + 1$ **(ans)**

(b) $(h^{-1}g)(x) = h^{-1}\left[(x-2)^2 + 3\right] = e^{(x-2)^2+3} + 1$

$(h^{-1}g)(x) = e^{(x-2)^2+3} + 1 = e^7 + 1$

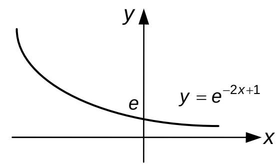
$(x-2)^2 + 3 = 7$

$(x-2)^2 = 4$

$x - 2 = \pm 2$

$x = 4$ or $x = 0$ (N.A since $x > 2$) **(ans)**

4.
(a) $D_h = (-\infty, 0]$, $R_h = [e, \infty)$.



Now, $D_g = (-1, \infty)$.

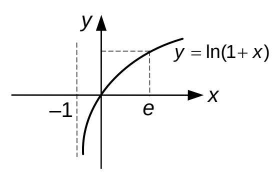
Thus, $R_h \subseteq D_g$.

gh exists.

$gh(x) = g\left(e^{-2x+1}\right) = \ln\left(1 + e^{-2x+1}\right)$

$D_{gh} = D_h = (-\infty, 0]$ **(ans)**

(b) $(-\infty, 0] \xrightarrow{h} [e, \infty) \xrightarrow{g} [\ln(1+e), \infty)$



$R_{gh} = [\ln(1+e), \infty)$ **(ans)**

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5. $f(x) = x^2 - 1$

The least value of α is 0.

Hence, $f(x) = x^2 - 1$, $x \geq 0$.

Let $y = x^2 - 1$

$x = \pm\sqrt{y+1}$

Since $x \geq 0$, $x = \sqrt{y+1}$

$f^{-1} : x \to \sqrt{x+1}$, $x \geq -1$. **(ans)**